

Spin-charge separation and Kondo effect in transport through a 1D Mott-Hubbard insulator.

V.V. Ponomarenko

Center of Physics, University of Minho, Campus Gualtar, 4710-057 Braga, Portugal

(Dated: January 21, 2013)

We study low energy spin and charge transport through a 1D Mott-Hubbard insulator of finite length L attached to Fermi liquid reservoirs characterized by different chemical potentials for electrons of opposite spin polarizations as it happens in quantum spin Hall insulators. We calculate the average currents (charge and spin) and their correlators and demonstrate how a transition induced by the reservoirs to the low energy Fermi liquid regime results in breakdown of the spin-charge separation, which is visible in the presence of the spin dependent voltages and a weak one electron scattering in the system. These calculations are carried out under assumption that the Hubbard gap $2M$ is large enough: $M > T_L \equiv v_c/L$ (v_c : charge velocity in the wire) and the scattering rate $\Gamma_s \ll T_L$. Relation of these results to Kondo dot transport in the Toulouse limit is also clarified.

PACS numbers: 71.10.Pm, 72.25.Mk, 73.40.Rw, 85.75.-d

Spin-charge separation in a 1D Mott-Hubbard insulator (MHI) known from the exact solution of the 1D Hubbard model [1] at half-filling and confirmed in experiments with quasi 1D materials [2, 3] can lead to an unusual effect if the insulator is used for transport between Fermi liquid (FL) reservoirs filled with electrons of opposite spin polarization up to different chemical potentials. Different spin-dependent chemical potentials can arise, in particular, in 2D quantum spin Hall insulators (QSHIs) [4], where transport is carried by pairs of edge states of up and down spin polarized electrons moving in opposite directions. Since in MHI the charge transfer realized by holons carrying one-electron charge e ($e = \hbar = 1$ below) is suppressed by a Hubbard gap $2M$, but spin carrying excitations, spinons, move freely, the perfect MHI serves as an ideal spin current transmitter. Any small difference in the spin dependent voltages $V_{\sigma}, \sigma = \pm$ reverses direction of one of the spin polarized currents in spite of large average difference between the source and drain chemical potentials. In the case of the QSHI reservoir the spin polarized tunneling currents can be found by measuring edge currents to the right and left from the junction with the MHI wire along the reservoir boundary. If there is only a single pair of edge states in the reservoir and the spin dependent voltages are applied anti-symmetrically, the ideal spin current transmittance through the MHI makes both these edge currents and, hence, conductance along the reservoir boundary zero, since the difference in the currents of the outgoing spin polarized edges is opposite to the difference of the incoming currents. This simple picture of the transport can be spoiled, however, by one electron processes which affect spin-charge separation in the MHI.

In this paper we consider effect of the FL reservoirs on low energy spin-charge separation in the MHI of finite length L : $M > T_L \equiv v_c/L$ (v_c : charge velocity in the wire) in the presence of one-electron impurity backscattering of low rate $\Gamma_s \ll M$. For low energy ($< T_L$) the

problem has been mapped [5] through a Duality Transform onto the model of a point scatterer with pseudospin imbedded in TLL and solved by fermionization. This model is also related to the Toulouse limit in Kondo dot transport [6] as discussed below. Its solution shows a Kondo-like resonance of conductance below the crossover energy $\Gamma = \Gamma_s + \Gamma_c$, which remains exponentially small ($\Gamma_c \approx \sqrt{T_L M} \exp\{-2M/T_L\}$ at half filling) in the absence of impurities due to rare tunneling of the condensate phase introduced by the reservoirs, but can grow up to $\sim M^2/E_F$ (E_F is Fermi energy in the wire) in the range of applicability of this model as the backscattering increases. We impose spin dependent voltages in this model to examine the spin-charge separation by calculating the average charge and spin currents and their zero frequency correlators and cross-correlator at temperature T . Each average current depends only on either charge $V_c = \sum_{\sigma} V_{\sigma}/2$ or spin $V_s = \sum_{\sigma} \sigma V_{\sigma}/2$ voltage combination, respectively. At the voltages or temperature above the crossover both currents display a very different behavior only weakly affected by the one electron scattering. In particular, the spin voltage $|V_s| > \Gamma$ indeed reverses direction of one of the spin polarized currents even for $|V_c| \gg |V_s|$. The currents cross-correlator asymptotically vanishes with increase of $\min |V_{c,s}|/\Gamma$ or T/Γ , while both diagonal correlators remain finite. This shows that the spin-charge separation in transport through the MHI occurs already in the Kondo regime at the energies less than $T_L (< M)$. Below the crossover the spin polarized combinations of the average currents and their correlators approach in the linear voltage limit the correspondent decoupled one-electron expressions [8] specified by the spin dependent voltages and a single parameter of the one-electron transmission. The latter is equal to $1/(1+\Gamma_s/\Gamma_c)$ due to exponential enhancement of the one-electron reflection Γ_s/Γ_c by the charge condensate tunneling.

Transport through a one-channel wire adiabatically attached to the left and right FL reservoirs is modeled by

a 1D system of electrons, whose pairwise interaction is local and switched on by function $\varphi(x) = \theta(x)\theta(L-x)$ inside the wire of length L . Applying bosonization we can describe the charge and spin density fluctuations $\rho_b(x, t) = e_b \partial_x \phi_b(x, t)/(\sqrt{2}\pi)$, $b = c, s$, respectively, with (charge and spin) bosonic fields $\phi_{c,s}$. Without impurities their Lagrangian symmetrical under the spin rotation reads

$$\mathcal{L} = \int dx \sum_{b=c,s} \left[\frac{v_b(x)}{2g_b(x)} \left\{ \left(\frac{\partial_t \phi_b(t, x)}{v_b \sqrt{4\pi}} \right)^2 - \left(\frac{\partial_x \phi_b(t, x)}{\sqrt{4\pi}} \right)^2 \right\} - E_F^2 U_b \varphi(x) \cos(2\mu_b x/v_b + \sqrt{2}\phi_b(t, x))/(\pi v_F) \right], \quad (1)$$

where $v_F(E_F)$ denotes the Fermi velocity(energy) in the channel. The parameter $\mu_c \equiv \mu$ varies the chemical potential inside the wire from its zero value at half-filling and $\mu_s = 0$. The constants of the forward scattering differ inside the wire $g_b(x) = g_b$ for $x \in [0, L]$ from those in the leads $g_b(x) = 1$, and an Umklapp scattering (backscattering) of the strength $U_c(U_s)$ is introduced inside the wire. The velocities $v_{c,s}(x)$ change from v_F outside the wire to some constants $v_{c,s}$ inside it. We can eliminate them rescaling the spacial coordinate x_{old} in the charge and spin Lagrangians of (1) into $x_{new} \equiv \int_0^{x_{old}} dy/v_{c,s}(y)$. As a result, the new coordinate will have an inverse energy dimension and the length of the wire becomes different for the charge mode $L \rightarrow 1/T_L$ and spin mode $L \rightarrow 1/T'_L$. Applying renormalization-group results of the uniform sin-Gordon model [9] at energies larger than T_L or T'_L we come to renormalized values of the parameters in (1). For the repulsive interaction when initially $g_s > 1 > g_c$, the constant U_s of backscattering flows to zero and g_s to 1, bringing the spin mode into the regime of the free TLL. The constant U_c of Umklapp process increases, reaching v_F/v_c at the energy cut-off corresponding to the mass of the soliton M if the chemical potential μ is less than M . Meanwhile, g_c flows to its free fermion value $g_c = 1/2$.

The spin-charge separation in Lagrangian (1) can be broken by additional one-electron scattering which entangles the spin and charge modes. We account for such a process by including a weak backscattering on a point impurity potential inside the wire $0 < x_0 < L$:

$$\mathcal{L}_p = -\frac{2E_F V_{imp}}{\pi} \cos\left(\frac{\phi_c(t, x_c)}{\sqrt{2}} + \varphi_0\right) \cos\left(\frac{\phi_s(t, x_s)}{\sqrt{2}}\right), \quad (2)$$

where $x_{c,s} = x_0/v_{c,s}$, $\varphi_0 \equiv \varphi + \mu x_c$ incorporates a phase of the scatterer φ . The amplitude V_{imp} of the impurity potential determines transmission coefficient as $1/(1 + V_{imp}^2)$.

Low energy model - An effective model for energies lower than some cut-off D' specified below has been derived [7] from the expression for the partition function \mathcal{Z} associated to the combined Lagrangian (1) and (2) following Schmid [10]. Without impurities the spin and

charge modes are decoupled. After integrating out ϕ_c in the reservoirs the charge mode contribution into \mathcal{Z} describes rare tunneling between neighbor degenerate vacua of the massive charge mode in the wire characterized by the quantized values of $\sqrt{2}\phi_c(\tau, x) + 2\mu x = 2\pi m$, m is integer. Variation of m by $a = \pm 1$ relates to passage of a (anti)soliton through the wire ((anti)-instanton in imaginary time τ). The tunneling amplitude may be found as Pe^{-s_0/T_L} , $s_0 = \sqrt{M^2 - \mu^2}$, ($\mu \ll M$) by mapping [5] onto a free fermionic model or instanton techniques [7]. The latter also evaluates the pre-factor $P = C \times \sqrt{D'}(s_0^3 T_L/M^2)^{1/4}$ with the constant C of the order of 1. The parameter D' is a high-energy cut-off to the long-time asymptotics of the kink-kink interaction: $F(\tau) = \ln\{\sqrt{\tau^2 + 1/D'^2}\}$ created by the reservoirs. It varies with μ and was estimated from the time scale of the instanton as $D' \simeq \sqrt{MT_L}$ at $\mu = 0$ and $D' \simeq (M/\mu)T_L$ if $\mu > T_L$. A crucial modification to this consideration produced by the impurity under the assumption $E_F V_{imp} \ll M$ ensues from the shift of the m -vacuum. Since it is equal to $(-1)^m E_F A \cos(\phi_s(\tau, x_s)/\sqrt{2})$, $A = 2V_{imp} \cos \varphi/\pi$ the neighbor vacua become non-degenerate. This can be accounted for by an auxiliary pseudospin variable with the correspondent Pauli matrix σ_3 . The energy splitting becomes an operator $\hat{\sigma}_3 A E_F \cos(\phi_s(\tau, x_0)/\sqrt{2})$ acting on the pseudospin, and every (anti)-instanton tunneling reverses the σ_3 -value with the Pauli matrix $\hat{\sigma}_1$. The interaction $F(\tau)$ coincides with the pair correlator of some bosonic field θ_c , whose evolution is ruled by the free TLL Lagrangian $\mathcal{L}_0[\theta_c]$ (uniform Lagrangian (1) with no interaction). Then, by ascribing factors $\exp(\mp i\theta_c(\tau_j, 0)/\sqrt{2})$ to the (anti)-instanton at the moment τ_j the functional integral for the partition function is reduced [5] to a standard Hamiltonian form $\mathcal{Z} = \text{cst} \times \text{Tr}\{e^{-\mathcal{H}/T}\}$ with

$$\mathcal{H} = \mathcal{H}_0[\phi_s(x)] + \mathcal{H}_0[\theta_c(x)] - A E_F \hat{\sigma}_3 \cos(\phi_s(x_s)/\sqrt{2}) - 2P e^{-s_0/T_L} \hat{\sigma}_1 \cos(\theta_c(0)/\sqrt{2}). \quad (3)$$

Here $\phi_s(x)$ and $\theta_c(x)$ are Schrödinger's bosonic operators related to the variables $\phi_s(\tau, x)$ and $\theta_c(\tau, x)$ of the functional integration. The free TLL Hamiltonian $\mathcal{H}_0[\phi_s(x)]$ ($\mathcal{H}_0[\theta_c(x)]$) is a function of the field $\phi_s(x)$ ($\theta_c(x)$) and its conjugated corresponding to the free TLL action $\mathcal{L}_0[\phi_s]$ ($\mathcal{L}_0[\theta_c]$), respectively. The model (3) is equivalent to the initial one (1) at low energy. It relates to a Point Scatterer with internal degree of freedom in TLL and is solved exactly through fermionization.

Fermionization - The Pauli matrices can be written as $\hat{\sigma}_\alpha = (-1)^{\alpha+1} \frac{i}{2} \sum_{\beta, \gamma} \epsilon^{\alpha, \beta, \gamma} \xi_\beta \xi_\gamma$ with Majorana fermions $\xi_{1,2,3}$ and antisymmetrical tensor $\epsilon: \epsilon^{123} = 1$. Since the interaction in (3) is point-like localized and its evolution involves only the appropriate time-dependent correlators, we can fermionize it making use of:

$$\psi_c(0) = -i\sqrt{\frac{D'}{2\pi}} \xi_3 e^{i\frac{\varphi_c(0)}{\sqrt{2}}}, \psi_s(0) = i\sqrt{\frac{E_F}{2\pi}} \xi_1 e^{i\frac{\phi_s(x_s)}{\sqrt{2}}}. \quad (4)$$

Here $\psi_{c,s}(0)$ is the $x = 0$ value of the charge (spin) fermionic field, respectively. These fields have linear dispersions taken after the related bosonic fields with momentum cut-offs (equal to the energy ones) D' and E_F , respectively. All states of negative energies are filled. Substitution of these fields into (3) produces a free-electron Hamiltonian:

$$\mathcal{H}_F = \sum_{a=c,s} \left\{ -i \int dx \psi_a^+ \partial_x \psi_a + \sqrt{\Gamma_a} [\psi_a^+(0) - \psi_a(0)] \xi \right\}, \quad (5)$$

where the interaction reduces to tunneling between the $\psi_{c,s}$ fermions and the Majorana one $\xi \equiv \xi_2$. Here $\Gamma_s = \frac{2E_F}{\pi} (\cos(\varphi) V_{imp})^2$ is the rate of the one-electron backscattering and the rate of the instanton tunneling is $\Gamma_c = 2\pi C^2 \sqrt{T_L s_0^3} e^{-2s_0/T_L} / M$.

Application of voltages V_σ between the left and right reservoirs due to the shift of their chemical potentials, in general, different for electrons of opposite spin polarizations $\sigma = \pm$ makes the system non-equilibrium and can be described with a gauge transformation $\phi_{c,s} \rightarrow \phi_{c,s} - \sqrt{2} V_{c,s} t$ in the real-time Lagrangian (1). Each instanton tunneling increases the condensate phase inside the wire and adds charge $\Delta\phi_c/(\sqrt{2}\pi) = 1$ to the left reservoir. The correspondent change in the energy of the system equal to V_c causes a shift $\theta_c/\sqrt{2} \rightarrow \theta_c/\sqrt{2} - V_c t$ in the cos argument in Eq.(3). Both transformations then can be accounted for with the shifts of the charge and spin fermion chemical potentials in Eq. (4) by $V_{c,s}$, respectively. Assuming below that both voltages are applied antisymmetrically and small enough, $|V_{c,s}| < T_L < M$, we neglect their effect on the other parameters in the fermionized Hamiltonian (3).

To find the charge and spin currents flowing through the channel we notice that each antiinstanton realized by $\psi_c(0)$ transfers charge 1 to the right reservoir with no transfer of spin. Then the charge current operator is $J_c = -i[\int dx \psi_c \psi_c^+, \mathcal{H}_F] = i\sqrt{\Gamma_c} [\psi_c^+(0) + \psi_c(0)] \xi \equiv j_c$. On the other hand, free passage of the spin current J_s through the channel is affected by backscattering due to the spin field interaction in Hamiltonian \mathcal{H} in Eq. (3). This makes the spin current equal to $J_s = V_s/\pi - j_s$, where the backscattered spin current operator can be found as $j_s = -\sqrt{2}\delta\mathcal{H}/\delta\phi_s = i\sqrt{\Gamma_s} [\psi_s^+(0) + \psi_s(0)] \xi$.

A crucial feature of the Hamiltonian \mathcal{H}_F in Eq. (5) is that its interaction and the currents it creates contain two different Majorana components $\eta_{a,\mp}(0)$ of each fermionic field ψ_a , respectively. These components are defined by $\psi_a(0, t) = (\eta_{a,+} - i\eta_{a,-})/\sqrt{2}$. Being orthogonal at the same time both the $\eta_{a,\pm}(t)$ components lose this property if taken at different times due to the applied voltages. Still, in Keldysh technique we need to use in non-equilibrium calculations both retarded and advanced cross-diagonal Green functions $g_{a,+,-}^{R,A}(t)$ of free Majorana fermions vanish. Only the cross-diagonal Green functions $g_{a,+,-}^{>,<}(t) = -g_{a,-,+}^{>,<}(t)$ are non-zero and equal to

$g_{a,+,-}^{>,<}(\omega) = [f((\omega - V_a)/T) - f((\omega + V_a)/T)]/2$ in the frequency representation, where f is the Fermi-distribution function. Then, from the Dyson equation the total cross-diagonal Green function

$$G_{a+,X}^{>,<} = i\sqrt{2\Gamma_a} (g_{a+,a-}^{>,<} - G_{\xi,X}^A + g_{a+,a-}^R - G_{\xi,X}^{>,<}) \quad (6)$$

reduces to the first product on the right-hand side, if the index X denotes ξ as the second field, or vanishes at all, if $X = b+$. The diagonal total Green function $G_{a+,a+}^{>}$ coincides with the free one.

Average currents - As follows from Eq. (6) the average of the operator $\langle j_a \rangle$ can be written as $\langle j_a \rangle = -i2\Gamma_a \int d\omega g_{a+,a-}^{>}(\omega) G_{\xi}^A(\omega)/(2\pi)$ and depends only on the correspondent voltage V_a , since the advanced Green function G_{ξ}^A does not contain information about voltages.

It is related to the free Green function $g_{\xi}^{A(R)} = 2/(\omega \mp i0)$ through the correspondent Dyson equation with the self-energy $\Sigma_{\xi}^{A(R)} = \pm i\Gamma$. Substitution of the expressions for both Green functions results in

$$\langle j_a \rangle = \frac{2\Gamma_a \Gamma}{\pi} \int d\omega \frac{f(\frac{\omega - V_a}{T}) - f(\frac{\omega + V_a}{T})}{\omega^2 + 4\Gamma^2}, \quad (7)$$

The average charge and spin currents $\bar{J}_{c,s}$ are equal to $\langle j_c \rangle$ and $V_s/\pi - \langle j_s \rangle$, respectively. At low temperature $T < \Gamma$ the integral in Eq. (7) converges to $\langle j_a \rangle = (2\Gamma_a/\pi) \arctan(V_a/(2\Gamma))$. Then the average spin polarized currents $\bar{J}_\sigma = (\bar{J}_c + \sigma \bar{J}_s)/2$ below the crossover Γ approach the one-electron expressions $\bar{J}_\sigma = DV_\sigma/(2\pi)$, where the spin independent transmittance $D = \frac{\Gamma_c}{\Gamma}$ demonstrates renormalization of the initial amplitude V_{imp} in Eq. (2) into $\sqrt{\Gamma_s/\Gamma_c}$ by the interaction inside the MHI. Above the crossover the charge current saturates at Γ_c while the spin current grows up as $V_s/\pi - \Gamma_s$. Therefore, for $|V_c| \gg |V_s|$ one of the conductances \bar{J}_σ/V_σ becomes negative as $|V_s| \gtrsim \pi\Gamma$ suggesting emergence of the spin-charge separation. Similarly, the separation may be expected at $T \gg \Gamma$. Indeed, the linear bias charge conductance is $G_c = \Gamma_c \psi'(1/2 + \Gamma/(\pi T))/(\pi^2 T)$, where $\psi'(x)$ is the derivative of the di-gamma function, $\psi'(1/2) = \pi^2/2$. The high temperature asymptotics of both conductances $G_c = \Gamma_c/(2T)$ and $G_s = 1/\pi - \Gamma_s/(2T)$ are defined by different Γ -parameters and independent of each other, and the condition $1 \gg |V_s/V_c| = G_c/G_s$ on the current reversing voltages is satisfied. We further examine correlators between the charge and spin currents.

Currents correlators - The zero-frequency current correlators $\delta^2 J_{ab}$ are related to the current operators correlators $\delta^2 j_{ab} = \int dt \exp[-i\omega t] \langle j_a(t) j_b(0) \rangle$, $\omega \rightarrow 0$ in the following way $\delta^2 J_{ab} = \pm \delta^2 j_{ab}$, where \pm stands for diagonal and cross-correlators, respectively. The diagonal spin current correlator at finite temperature also includes[11] additional terms $\delta^2 J_{ss} = 2T/\pi - 4T\partial V_s/\partial j_s + \delta^2 j_{ss}$. Appearance of these terms may be easily understood recalling that the fluctuation-dissipation theorem claims non-

zero current fluctuations even in the absence of backscattering at non-zero temperature.

Substituting expressions for the current operators $j_{a,b}$ into their correlator and then splitting the correlator into pair-wise correlators of the Majorana fields by applying Wick's theorem we find that

$$\delta^2 j_{ab} = -2\sqrt{\Gamma_a \Gamma_b} \int \frac{d\omega}{2\pi} [G_{a+, \xi}^>(\omega) G_{a+, \xi}^<(\omega) + G_{a+b+}^>(-\omega) G_{\xi}^>(\omega)] \equiv I_{ab}^{(1)} + \delta_{a,b} I_a^{(2)} \quad (8)$$

Then the cross-correlator of the two currents equal to $-I_{cs}^{(1)}$ in Eq. (8) follows from Eq. (6) as

$$\delta^2 J_{cs} = -\frac{4\Gamma_c \Gamma_s}{2\pi} \int d\omega [G_{\xi}^A(\omega)]^2 g_{c+-}^>(\omega) g_{s+-}^<(\omega) = -\frac{4\Gamma_c \Gamma_s}{\pi} \int d\omega \frac{\prod_{a=c,s} [f(\frac{\omega-V_a}{T}) - f(\frac{\omega+V_a}{T})]}{(\omega - 2i\Gamma)^2}. \quad (9)$$

It is vanishing with increase of both voltages or temperature under the integral in Eq. (9) due to the analytical structure of the advanced Green function. In particular, in the limit of $|V_{c,s}|/T \gg 1$ it takes the form

$$\delta^2 J_{cs} = \frac{4\Gamma_c \Gamma_s}{\pi} \frac{\text{sign}(V_c V_s) V}{[4\Gamma^2 + V^2]}, \quad V = \min_{a=c,s} |V_a|, \quad (10)$$

which approaches zero $\propto \Gamma_c \Gamma_s / V$ as V becomes much larger than 2Γ . In general, the integral in Eq. (9) is expressed in terms of derivatives of the di-gamma function. This expression shows that at high temperature $T \gg 2\Gamma$ the cross-correlator is vanishing as $\delta^2 J_{cs} = |\psi^{(2)}(1/2) + \psi^{(4)}(1/2)| \Gamma_c \Gamma_s V_c V_s / (12\pi^5 T^3)$ if $1 \gg \max |V_{c,s}|/T$.

The diagonal correlators of the charge and spin currents also include the second term in Eq. (8). To find it we notice that the Green function $G_{\xi}^>$ of the single Majorana operator reduces to $G_{\xi}^> = G_{\xi}^R \Sigma_{\xi}^> G_{\xi}^A$ with the self energy $\Sigma_{\xi}^> = -i \sum_{a,\pm} \Gamma_a f((-\omega \pm V_a)/T)$ and $g_{a++}^>(-\omega) = -i \sum_{\pm} f((\omega \pm V_a)/T)$. Substitution of these expressions into Eq. (8) gives us the diagonal zero-frequency correlator of the charge current in the following form

$$\delta^2 J_{cc} = \frac{2\Gamma_c}{\pi} \int d\omega \frac{[f(\frac{\omega+V_c}{T}) + f(\frac{\omega-V_c}{T})]}{\omega^2 + 4\Gamma^2} \times \sum_{a=c,s} \Gamma_a \left[f(\frac{-\omega+V_a}{T}) + f(\frac{-\omega-V_a}{T}) \right] + I_{cc}^{(1)} \quad (11)$$

and the similar expression for correlator of the spin backscattered current $\delta^2 j_{ss}$ by exchanging the "c" and "s"-indexes. At zero temperature the latter correlator coincides with $\delta^2 J_{ss}$ and hence both shot noises of the charge and spin current ($a = c, s$) rise from Eq. (11) as follows:

$$\delta^2 J_{aa} = \left(\frac{2\Gamma_a^2}{\Gamma\pi} \arctan\left(\frac{|V_a|}{2\Gamma}\right) + \frac{2\Gamma_c \Gamma_s}{\pi\Gamma} \times \arctan\left(\frac{\max\{|V_b|\}_{b=c,s}}{2\Gamma}\right) - \frac{4\Gamma_a^2 |V_a|}{\pi(V_a^2 + 4\Gamma^2)} \right) \quad (12)$$

In the low voltage limit both expressions coincide $\delta^2 J_{cc} = \delta^2 J_{ss} = (1/\pi) D(1-D) \max\{|V_b|\}_{b=c,s}$. Combined with the low voltage cross-correlator in Eq. (10) they give shot noise of the two spin polarized currents [8] in the one-electron form $\delta^2 J_{\sigma\sigma'} = (\delta_{\sigma\sigma'}/\pi) D(1-D) |V_{\sigma}|$.

As both voltages increase above the crossover the shot noise in the two currents grows and saturates at two different values $\delta^2 J_{aa} = \Gamma_a$. Together with demonstrated before suppression of the cross-correlator of the currents this confirms the spin-charge separation above the crossover but yet below T_L . Similarly, the charge current noise $\delta^2 J_{cc} = \Gamma_c [1 + 2\psi^{(2)}(1/2)\Gamma/(\pi^3 T)]$ remains finite at high-temperature, while the spin noise grows linearly $\delta^2 J_{ss} = 2T/\pi - \Gamma_s = 2T\Gamma_s$.

Toulouse limit in Kondo dot transport - This model has been written [6] as a formal generalization of the physical model of Kondo dot transport. It describes tunneling between two branches, $a = r, l$, of 1D chiral fermions $\psi_{a,\sigma}(x)$ carrying spin σ and propagating in the right and left reservoirs, respectively, with the tunneling Hamiltonian

$$\mathcal{H}_T = \sum_{a,b=r,l} \sum_{\sigma,\sigma',\gamma} J_{\gamma}^{a,b} \hat{\tau}_{\gamma} \psi_{a,\sigma}^+(0) \sigma_{\sigma,\sigma'}^{\gamma} \psi_{b,\sigma'}(0), \quad (13)$$

where $\hat{\tau}_{\gamma}$ are the Pauli matrices for the Kondo dot spin. The choice of the parameters $J_z^{a,a} = 2\pi$, $J_z^{r,l} = 0$, $J_{x,y}^{a,b} = J_{\perp}^{a,b}$ corresponds to the Toulouse limit model solvable through bosonization and Emery-Kivelson rotation. Under additional restriction $J_{\perp}^{r,r} = -J_{\perp}^{l,l}$ its Hamiltonian takes the form of \mathcal{H} in Eq. (3) with the field ϕ_s substituted by its dual θ_s , which describes tunneling of the unit of spin instead of its backscattering, and further coincides with \mathcal{H}_F in Eq. (5) after the fermionization with $\Gamma_c = E_F [J_{\perp}^{r,l}]^2 / (8\pi)$, $\Gamma_s = E_F [J_{\perp}^{r,r}]^2 / (8\pi)$. Both operators $j_{c,s}$ now describe tunneling of the charge and the spin. Interchanging the direct and backscattered spin currents in the above calculation we apply their results to the Toulouse limit model. In particular, the low energy linear voltage dependence of the average charge and spin currents is defined by different transmittances equal to D and $1-D$, respectively. This feature can not be accounted for in the one-electron transport model. Indeed, in this model the spin cross-diagonal transmission and reflection coefficients become $\pm(D-1/2)$, and one of them would be negative unless $D = 1/2$. This indicates independent tunneling of spinons and break of the low energy electronic FL description.

In summary, we have shown that in spite of influence of the FL reservoirs and a weak one-electron scattering the essential properties of the spin-charge separation in transport through a MHI of finite length appear if either T or both $|V_{c,s}|$ are above a crossover energy ($\ll T_L$), below which, however, the transport becomes one-electron.

-
- [1] E.H.Lieb and F.Y.Wu, Phys. Rev. Lett.**20**, 1445 (1968).
 - [2] B.J.Kim *et al*, Nature Phys. **2**, 397 (2006).
 - [3] R. Neudert *et al*, Phys. Rev. Lett. **81**, 657 (1998).
 - [4] C.L. Kane and E.J. Mele, Phys. Rev. Lett. **95**, 226801 (2005); B.A.Bernevig and S.-C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006); Z. Hasan and C.L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
 - [5] V. V. Ponomarenko and N. Nagaosa, Solid State Commun. **114**, 9 (2000); Phys. Rev. Lett. **81**, 2304 (1998).
 - [6] A. Schiller and S. Hershfield, Phys. Rev. B **58**, 14978 (1998); T. L. Schmidt, A. O. Gogolin, and A. Komnik, Phys. Rev. B **75**, 235105 (2007).
 - [7] V. V. Ponomarenko and N. Nagaosa, Phys. Rev. Lett. **83**, 1822 (1999).
 - [8] G.B. Lesovik, JETP Lett. **49**, 592 (1989); V.A. Khlus, Sov. Phys. JETP **66**, 1243 (1987).
 - [9] Solyom, Adv. Phys. **31**, 293 (1979).
 - [10] A. Schmid, Phys. Rev. Lett. **51**, 1506 (1983).
 - [11] V. V. Ponomarenko and N. Nagaosa, Phys. Rev. B **60**, 16865 (1999); Solid State Commun. **110**, 321 (1999).